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PROJECTILE IMPACT ON AN INFINITE, VISCOPLASTIC PLATE†

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Abstract—An analysis is presented for the problem of a rigid/viscoplastic infinite plate subjected to normal projectile impact. The solution is first obtained in Laplace transform space for the finite clamped plate and then limiting conditions and numerical inversion methods are used to get the solution for the infinite plate in the time domain. The results are compared with experimental data for projectile impact on mild steel and aluminum alloy plates. The theory presents a useful analytical method for the evaluation of the response of large plates composed of strain rate dependent material subjected to projectile impact.

INTRODUCTION

THIS paper presents a solution to a problem in the area of projectile impact on thin plates using the theory of rate sensitive plasticity as a model for the response of the plate material. Problems of the dynamic plastic behavior of structures are of considerable technical importance but analytical solutions are confined largely to very simple physical situations such as circular plates symmetrically loaded, and generally within the framework of the theory of rate independent rigid plastic materials. Thus Hopkins [1] treated the motion of a rigid perfectly plastic plate of material obeying the Tresca yield condition and flow rule subject to an accelerative velocity, imposed on the plate by a rigid cylindrical punch. No dynamic interaction between projectile and plate occurred in this problem since the motion of the punch was specified *a priori*. Chulay [2] studied the problem of a concentrated dynamic load on a rigid perfectly plastic plate exhibiting a piecewise linear yield condition and using a single propagating yield hinge in the model. A solution was found only for the case of constant transverse velocity and solutions for other types of loading or for work-hardening materials were found not to exist within the limitations of this model.

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A fairly large number of studies involving plates subjected to impulsive loading as opposed to projectile impact has been published, again within the framework of rigid perfectly plastic material behavior. The simply supported plate subject to an ideal impulse and to a rectangular pulse was examined by Wang [3] and by Hopkins and Prager [4], respectively. Florence has obtained the response of a clamped plate to a rectangular pulse over the entire surface (Florence [5]) and over a central region (Florence [6]). In all of the above cases a Tresca yield condition was assumed.

Experimental studies corresponding to the above theoretical analyses provide results which are considerably different from the theoretical predictions (see, for example, Florence [7]) the differences generally being attributed to the effects of membrane forces which are neglected in the analyses. However, all these solutions were based on the assumption that the yield stress of the material was independent of strain rate. The plastic behavior of most metals is sensitive to strain rate, mild steel particularly so; aluminum, although less so, is also subject to rate effects. Furthermore, the flow rule associated with the Tresca yield condition leads to velocity fields which are unrealistic as a result of the piecewise constant direction required of the strain rate vector. In a recent paper Wierzbicki and Florence [8] have shown that the differences between theory and experiment can be significantly reduced by inclusion of rate effects in the lower strain rate regime, whereas in the high strain rate domain membrane effects appear to have a stronger influence.

In the present paper the behavior of the plate is analyzed on the basis of a rigid visco-plastic material obeying a quasi-static yield condition of the von Mises type and its associated flow rule, and uses the method of linearization by Kelly and Wierzbicki [9] to obtain a solution to the problem of projectile impact on a clamped circular plate for the same viscoplastic material model and yield condition. This solution incorporated the dynamic interaction of the projectile and the plate, the impact force not being specified in advance but being part of the solution. The resulting displacement of the plate was presented in the form of an infinite sum of eigenfunctions. This solution was applied to an experimental study by Kelly and Wilshaw [10] and it was found that the series converged rapidly and very few terms were needed to give results in good accord with the experimental data. However, the plates used in [10] were fairly thick plates having a diameter to thickness ratio in the range 10–25. In the present case when this solution was used to predict plate deformations it was found that the convergence of the series was extremely slow and an improved solution was needed. The reason for the slow convergence of the series was the very large diameter to thickness ratio of the plate and the fact that the boundary of the plate was so distant from the point of impact that it did not influence the plastic behavior which was localized near the point of impact. Thus it was felt to be useful to develop, as an alternative to the method proposed in [9], a method of solution which would have special applicability to plates of large diameter to thickness ratio and this solution is presented in this paper. The solution obtained here is strictly valid only for circular plates of infinite radius but it is clear from both theoretical and experimental results that plastic deformation does not develop near the edges of the plates. This was verified by showing that the plastic deformation of a four foot square plate was identical to that of a 15 in. diameter circular plate of the same thickness and subject to the same impact. A comparison of the terminal displacement profiles of a $14\frac{1}{4}$ in. diameter clamped plate and a 4×4 ft freely-suspended plate, both composed of 2024-0 aluminum and 0·050 in. thick, struck by a $\frac{1}{2}$ in. diameter steel sphere at velocities of 396 and 397 ft/sec is shown in Fig. 1 and attests to the validity of this hypothesis.

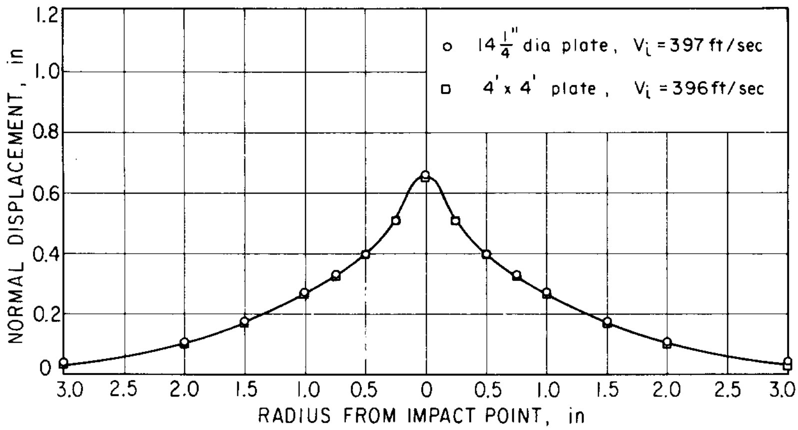

FIG. 1. Terminal displacement profiles of a $14\frac{1}{4}$ in. diameter clamped plate and a 4 × 4 ft freely-suspended plate, both composed of 2024-0 aluminum and 0·050 in. thick when struck by a $\frac{1}{2}$ in. diameter steel sphere.

The solution presented here supplements that obtained in [9] and does not duplicate it. In fact by an obvious procedure the two can be combined to provide solutions which are convenient in computation over the entire range of diameter to thickness ratios.

THEORETICAL CONSIDERATIONS

(i) *Viscoplastic stress strain relations*

The uniaxial behavior of a viscoplastic material may be approximately represented by a relationship between stress σ and plastic strain rate $\dot{\varepsilon}$ of the form

$$\dot{\varepsilon} = \frac{2\gamma}{\sqrt{3}}(\sigma - \sigma_0)/\sigma_0 \tag{1}$$

where σ_0 is the static yield stress in simple tension and $1/\gamma$ is the viscoplastic relaxation time of the material.

Generalizations of equation (1) to non-uniaxial stress have been considered by a number of authors; in particular Perzyna [11] and Craggs [12] from different basic premises have obtained several forms of multiaxial stress strain relation. The simplest form which reduces to equation (1) for uniaxial loading, and which will be used here is

$$\dot{\varepsilon}_{ij} = \gamma \frac{(s_{kl}s_{kl}/2)^{\frac{1}{2}} - k}{k} \cdot \frac{s_{ij}}{(s_{kl}s_{kl}/2)^{\frac{1}{2}}} \tag{2}$$

applicable when $\frac{1}{2}s_{kl}s_{kl} \geq k^2$. In equation (2) $\dot{\varepsilon}_{ij}$ is the strain rate tensor and s_{ij} the stress deviator, $1/\gamma$ is as before the relaxation time and $k = \sigma_0/\sqrt{3}$ the static yield stress in simple shear. The material is taken as incompressible for plastic deformations and the elastic deformations are neglected.

The physical basis of equation (2) is the assumption that the material obeys the von Mises criterion

$$\tfrac{1}{2}s_{ij}s_{ij} = k^2$$

and its associated flow rule for static deformations and an expanded von Mises yield condition and associated flow rule for dynamic deformations, and further the viscosity of the material requires that the strain rate depend on the difference between the expanded and the static yield condition. The expansion of the yield condition at any time and any location in the body is given by squaring both sides of equation (2) leading to

$$(\tfrac{1}{2}s_{ij}s_{ij})^{\frac{1}{2}} = k\left[1+\frac{1}{\gamma}(\tfrac{1}{2}\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij})^{\frac{1}{2}}\right]$$

The flow rule asserts that the strain rate should be normal to the yield surface at any time. In the nine-dimensional space of the stress deviator the yield condition is a hypersphere and the requirement of the flow rule is met by noting that $s_{ij}/(s_{kl}s_{kl})^{\frac{1}{2}}$ is a radial unit vector in this space.

(ii) *Governing equations of thin plate theory*

In setting up the governing equations of the viscoplastic plate all quantities are assumed to be functions only of r the distance measured from the plate center and of the time t. The surface tractions p or p_0 are taken positive in the direction of positive transverse displacements of the middle surface. The velocity of points of the middle surface is $v(r, t)$. The plate radius is R, the thickness is $2h$ and the mass density per unit of area of the middle surface is μ.

The constitutive relations of equation (2) when written in terms of the radial and circumferential moments M_r and M_ϕ and the corresponding curvature rates $\dot{k}_r$ and $\dot{k}_\phi$ take the form (see for example [13])

$$\dot{k}_r = \frac{\sqrt{3}\gamma}{2h}\left(1-\frac{M_0}{\sqrt{(M_r^2-M_rM_\phi+M_\phi^2)}}\right)\frac{2M_r-M_\phi}{M_0}$$

$$\dot{k}_\phi = \frac{\sqrt{3}\gamma}{2h}\left(1-\frac{M_0}{\sqrt{(M_r^2-M_rM_\phi+M_\phi^2)}}\right)\frac{2M_\phi-M_r}{M_0}$$

where $M_0 = \sigma_0 h^2$.

The kinematics of the deformation require that the rates of curvature $\dot{k}_r$ and $\dot{k}_\phi$ be related to the velocity v through

$$\dot{k}_r = -v_{,rr}; \qquad \dot{k}_\phi = -v_{,r}/r \tag{3}$$

Following the method described in [9] we linearize the constitutive relations by assuming that the stress trajectory in the nine dimensional space of the stress deviator of any particle is a straight line. Thus the quantity $s_{ij}/(s_{kl}s_{kl})^{\frac{1}{2}}$ = const. and the stress strain rate relation becomes

$$\dot{\varepsilon}_{ij} = \gamma(s_{ij}-\bar{s}_{ij})/k \tag{4}$$

where $\bar{s}_{ij}$ is the state of stress on the surface $\tfrac{1}{2}s_{ij}s_{ij} = k^2$. This condition is satisfied strictly only at the center of the plate where $k_r = k_\phi$ and at the edge of the plate but deviations from the straight line may be small at intermediate points. The equations relating moments and curvature rates corresponding to (4) are

$$\dot{k}_r = \frac{\sqrt{3}\gamma}{2h}(2M_r - M_\phi - (2\overline{M}_r - \overline{M}_\phi))/M_0$$

$$\dot{k}_\phi = \frac{\sqrt{3}\gamma}{2h}(2M_\phi - M_r - (2\overline{M}_\phi - \overline{M}_r))/M_0$$

where now $\overline{M}_r$ and $\overline{M}_\phi$ are moments satisfying the initial yield condition $M_r^2 - M_r M_\phi + M_\phi^2 = M_0^2$.

Using the above relations and equation (3) the governing equation of the plate velocity v is

$$\nabla^4 v = \frac{3\sqrt{3}\gamma}{4hM_0}\left\{\frac{1}{r}[(r\overline{M}_r)_{,r} - \overline{M}_\phi]_{,r} + p - \mu v_{,t}\right\}$$

The pressure p in this case is zero except of the region of contact of the plate and the projectile and given by

$$2\pi \int pr\,\mathrm{d}r = -mv_{,t}|_{r=0}$$

where m is the mass of the projectile.

The quantity

$$-\frac{1}{r}[(r\overline{M}_r)_{,r} - \overline{M}_\phi]_{,r}$$

is a pressure distribution and corresponds to that at the static collapse condition of a rigid perfectly plastic plate obeying the von Mises yield condition. We denote the static pressure distribution corresponding to the dynamic loading p by p_0. In this case p is a concentrated point load at $r = 0$ and p_0 is a concentrated point load of magnitude $4\pi M_0/\sqrt{3}$. The impacting mass in this case is taken to be rigid, of negligible radius and travelling with velocity V_i.

The governing partial differential equation for the clamped viscoplastic plate subjected to central projectile impact based on small deflections and bending action only is thus

$$\nabla^4 v + \frac{3\sqrt{3}\gamma\mu}{4hM_0} v_{,t} = 0$$

subject to initial conditions

$$v(0,0) = V_i; \qquad v(r,0) = 0, \qquad r \neq 0$$

and boundary conditions

$$v(R,t) = 0, \qquad v_{,r}(R,t) = 0$$

and

$$\lim_{r\to 0} 2\pi r \frac{4hM_0}{3\sqrt{3}\gamma}(\nabla^2 v)_{,r} = -\frac{4\pi M_0}{\sqrt{3}} - mv_{,t}\bigg|_{r=0}$$

where v is the plate velocity and a comma indicates differentiation with respect to the subscript variable.

(iii) *Solution in transform space*

The resulting equations in Laplace transform space can then be written as

$$\nabla^4\bar{v}(r,s)+\alpha^4 s\bar{v}(r,s) = 0 \tag{5}$$

subject to

$$\bar{v}(R,s) = 0, \qquad \bar{v}_{,r}(R,s) = 0 \tag{6}$$

and

$$\lim_{r\to 0} 2\pi r(\nabla^2\bar{v}(r,s))_{,r} = -\bar{Q}(s) \tag{7}$$

where

$$\alpha^4 = \frac{3\sqrt{3}\gamma\mu}{4hM_0}, \qquad \bar{Q}(s) = \frac{\alpha^4}{\mu}\left[\frac{P}{s}+m(s\bar{v}(0,s)-V_i)\right] \qquad \text{and} \qquad P = \frac{4\pi M_0}{\sqrt{3}}$$

The barred quantities are the Laplace transformed variables.

The solution of equation (5) can be written in terms of Kelvin functions [14], as

$$\bar{v}(r,s) = A(s)\,\mathrm{ber}(\alpha s^{\frac{1}{4}}r)+B(s)\,\mathrm{bei}(\alpha s^{\frac{1}{4}}r)+C(s)\,\mathrm{ker}(\alpha s^{\frac{1}{4}}r)+D(s)\,\mathrm{kei}(\alpha s^{\frac{1}{4}}r) \tag{8}$$

where the parameters A, B, C and D are determined from the boundary conditions. Now in the limit as $x \to 0$, the behavior of the Kelvin functions is $\mathrm{ber}\,x \to 1$, $\mathrm{bei}\,x \to 0$, $\mathrm{ker}\,x \to -\infty$ and $\mathrm{kei}\,x \to -\pi/4$. Therefore, requiring boundedness on $\bar{v}(r,s)$ as $r \to 0$ implies that $C(s)$ must vanish.

In dealing with the boundary condition given by equation (7), it is noted that in the limit as $x \to 0$, the behavior of the derivatives of the Kelvin functions is $\mathrm{ber}'x$, $\mathrm{ber}''x$, $\mathrm{ber}'''x$, $\to 0$; $\mathrm{bei}'x$, $\mathrm{bei}'''x \to 0$; but $\mathrm{bei}''x \to \frac{1}{2}$ and the derivatives of kei x also do not vanish in the limit. Therefore, in evaluating equation (7) for the limiting case it is necessary just to consider the kei terms and their derivatives and terms containing $\mathrm{bei}''x$.

Letting $q = \alpha s^{\frac{1}{4}}$, the following relations for the derivatives of the necessary Kelvin functions can be established:

$$\mathrm{kei}(qr)_{,r} = q\,\mathrm{kei}'(qr)$$

$$\mathrm{kei}(qr)_{,rr} = q^2\left(-\frac{\mathrm{kei}'(qr)}{qr}+\mathrm{ker}(qr)\right)$$

$$\mathrm{kei}(qr)_{,rrr} = q^3\left(\mathrm{ker}'(qr)+\frac{2}{q^2r^2}\,\mathrm{kei}'(qr)-\frac{1}{qr}\,\mathrm{ker}(qr)\right)$$

$$\mathrm{bei}(qr)_{,rr} = q^2\,\mathrm{bei}''(qr)$$

Substituting into the derivative of the Laplacian occurring in equation (7) and cancelling like terms gives

$$r\bar{v}_{,rrr}+\bar{v}_{,rr}-\frac{1}{r}\bar{v}_{,r} = B(s)q^2\,\mathrm{bei}''(qr)+D(s)q^2(qr)\,\mathrm{ker}'(qr)$$

Now as $x \to 0$, $\mathrm{ker}'x$ approaches $-2/x\,\mathrm{ber}\,x$ and $\mathrm{ber}\,x$ approaches 1. Therefore $\lim\,(qr)\,\mathrm{ker}'(qr) = -2$. Also, as previously noted, $\mathrm{bei}''(qr)$ approaches $\frac{1}{2}$ as $(qr) \to 0$. Thus,

$$\lim_{r\to 0}\left(r\bar{v}_{,rrr}+\bar{v}_{,rr}-\frac{1}{r}\bar{v}_{,r}\right) = \tfrac{1}{2}B(s)\alpha^2 s^{\frac{1}{2}}-2D(s)\alpha^2 s^{\frac{1}{2}} \tag{9}$$

so that equation (7) becomes

$$B(s) \,.\, \pi\alpha^2 s^{\frac{1}{2}} - D(s) \,.\, 4\pi\alpha^2 s^{\frac{1}{2}} = -\bar{Q}(s) \tag{10}$$

The two boundary conditions of equation (6) give

$$A(s)\,\mathrm{ber}(\alpha s^{\frac{1}{2}} R) + B(s)\,\mathrm{bei}(\alpha s^{\frac{1}{2}} R) + D(s)\,\mathrm{kei}(\alpha s^{\frac{1}{2}} R) = 0 \tag{11}$$

$$A(s)\,\mathrm{ber}'(\alpha s^{\frac{1}{2}} R) + B(s)\,\mathrm{bei}'(\alpha s^{\frac{1}{2}} R) + D(s)\,\mathrm{kei}'(\alpha s^{\frac{1}{2}} R) = 0 \tag{12}$$

The three equations (10)–(12) can be solved simultaneously for the coefficients $A(s)$, $B(s)$ and $D(s)$. Thus, the transform solution is given by equation (8) with $C(s) = 0$ and the remaining coefficients known. The analytical inversion of this transform solution for the clamped finite plate does not appear to be feasible. However, if R becomes very large, asymptotic results for the Kelvin functions may be employed to derive a solution for the infinite plate.

The asymptotic expansions of the Kelvin functions for large arguments, [14], show that the Kelvin functions for $x \to \infty$ behave as

$$\mathrm{ber}\,x \sim \frac{e^{x/\sqrt{2}}}{\sqrt{(2\pi x)}}\cos\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8}\right) \qquad \mathrm{ber}'\,x \sim \frac{e^{x/\sqrt{2}}}{\sqrt{(2\pi x)}}\cos\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8}\right)$$

$$\mathrm{bei}\,x \sim \frac{e^{x/\sqrt{2}}}{\sqrt{(2\pi x)}}\sin\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8}\right) \qquad \mathrm{bei}'\,x \sim \frac{e^{x/\sqrt{2}}}{\sqrt{(2\pi x)}}\sin\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8}\right)$$

$$\mathrm{kei}\,x \sim \sqrt{\left(\frac{\pi}{2x}\right)}\, e^{-x/\sqrt{2}}\sin\left(\frac{\pi}{\sqrt{2}} + \frac{\pi}{8}\right) \quad \mathrm{kei}'\,x \sim \sqrt{\left(\frac{\pi}{2x}\right)}\, e^{-x/\sqrt{2}}\sin\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8}\right)$$

Using these expansions to determine the behavior of $A(s)$, $B(s)$ and $D(s)$ as $R \to \infty$, it is found that

$$A(s) \to 0, \qquad B(s) \to 0, \qquad D(s) \to \frac{\bar{Q}(s)}{4\pi\alpha^2 s^{\frac{1}{2}}}$$

Therefore, the transform solution for the infinite plate is

$$\bar{v}(r, s) = \frac{\bar{Q}(s)}{4\pi\alpha^2 s^{\frac{1}{2}}}\,\mathrm{kei}(\alpha s^{\frac{1}{2}} r) \tag{13}$$

Now recall that $\bar{Q}(s)$ contains a $\bar{v}(0, s)$ term. As $r \to 0$, the solution of equation (13) for $\bar{v}(0, s)$, noting that $\mathrm{kei}(0) = -(\pi/4)$, is

$$\bar{v}(0, s) = \frac{V_i}{s^{\frac{1}{2}}(b + s^{\frac{1}{2}})} - \frac{P/m}{s\,.\,s^{\frac{1}{2}}(b + s^{\frac{1}{2}})} \tag{14}$$

where

$$b = \frac{16\mu}{\alpha^2 m}$$

Substituting this result back into equation (13) gives the complete viscoplastic transformed solution for projectile impact on an infinite plate as

$$\bar{v}(r, s) = \frac{4}{\pi}\left[\frac{P/m}{s\,.\,s^{\frac{1}{2}}(b + s^{\frac{1}{2}})} - \frac{V_i}{s^{\frac{1}{2}}(b + s^{\frac{1}{2}})}\right]\mathrm{kei}(\alpha s^{\frac{1}{2}} r) \tag{15}$$

Although the possibility of obtaining an inverse in closed form appears to be remote, a numerical inversion based on a scheme developed by Dubner and Abate [15] was obtained at points away from $r = 0$.

An analytical solution at $r = 0$ is easily obtained by inverting equation (14), yielding

$$v(0, t) = \left(V_i - \frac{P}{mb^2}\right) e^{b^2 t} \operatorname{erfc}(bt^{\frac{1}{2}}) + \frac{P}{mb^2}\left[1 - 2b\left(\frac{t}{\pi}\right)^{\frac{1}{2}}\right] \tag{16}$$

Integrating this result gives the displacement–time relation at the center of the plate as

$$w(0, t) = \left(V_i - \frac{P}{mb^2}\right)\left[\frac{1}{b^2} e^{b^2 t} \operatorname{erfc}(bt^{\frac{1}{2}}) + \frac{2}{b}\left(\frac{t}{\pi}\right)^{\frac{1}{2}} - \frac{1}{b^2}\right] + \frac{P}{mb^2} t - \frac{4P}{3mb\sqrt{\pi}} t^{\frac{3}{2}} \tag{17}$$

Similarly, differentiation of equation (16) gives the plate deceleration with time as

$$a(0, t) = \left(V_i - \frac{P}{mb^2}\right)\left[b^2 e^{b^2 t} \operatorname{erfc}(bt^{\frac{1}{2}}) - \frac{b}{\sqrt{\pi t}}\right] - \frac{P}{mb\sqrt{\pi t}} \tag{18}$$

This last result indicates an infinite deceleration at the instant of impact. It appears that the reason for this is inherent in the choice of a rigid/viscoplastic material model and the requirement of an instantaneous velocity rise of $v = V_i$ at the plate center upon impact. In reality, because of Hertz contact and indentation effects, there is a rise time associated with the plate center reaching the velocity of the projectile. The viscoplastic theory is also singular in that it does not reduce exactly to the rigid/plastic theory for $\gamma \to \infty$, corresponding to no strain rate sensitivity.

Because of the use in the solution of a constant collapse load P, the velocity relation will continue to decrease monotonously with increasing time. However, the result has physical meaning only up to the time when the central plate velocity has reached zero. The time, t_f, at which this occurs is determined from the relation obtained by setting $v(0, t_f) = 0$ in equation (16):

$$\left(V_i - \frac{P}{mb^2}\right) e^{b^2 t_f} \operatorname{erfc}(bt_f^{\frac{1}{2}}) + \frac{P}{mb^2}\left[1 - 2b\left(\frac{t_f}{\pi}\right)^{\frac{1}{2}}\right] = 0 \tag{19}$$

DISCUSSION

The variation of the permanent central deflection of the plates with impact velocity as predicted by the viscoplastic theory and as observed is presented in Figs. 2 and 3 for 2024-0 aluminum and mild steel, respectively. A value of $\gamma = 1000\ \text{sec}^{-1}$ was used for the aluminum which is only slightly strain-rate sensitive. Values of γ of 400, 100 and 50 sec^{-1} were employed in the calculation for the mild steel plates; the first of these values having been deduced from experimental results on a similar mild steel by Kelly and Wilshaw [10]. Furthermore, the value of M_0 employed in the present calculations for the latter material was not derived from the value of the measured static yield stress. It is well known that the dynamic yield stress for mild steel is a highly non-linear function of strain rate and rises rapidly with this parameter in the range $0 \leq \dot{\varepsilon} \leq 10\ \text{sec}^{-1}$, but much more slowly above a value of $100\ \text{sec}^{-1}$. The work of a number of authors concerned with this topic has been

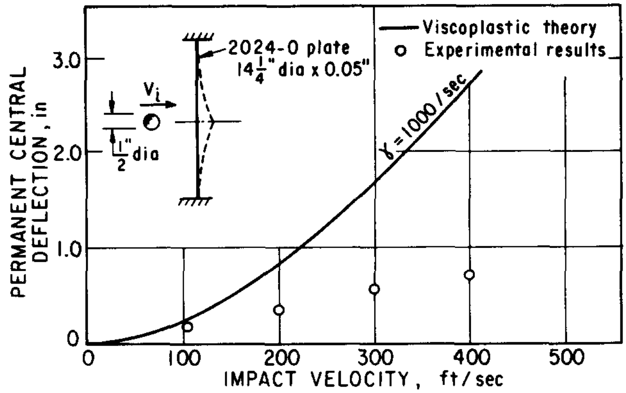

FIG. 2. Comparison of viscoplastic infinite plate theory with experimental results for projectile impact on a 2024-0 aluminum plate (Ref. [17]).

compiled by Symonds [16] and a representative stress–strain rate curve using this information has been published by Wierzbicki and Florence [8]. From the strain histories presented in Ref. [7], it was estimated that the average strain rate of the initial transient in the case of the steel plates was in the range from 100 to 200 $\sec^{-1}$. For this region, the tangent to the stress–strain rate curve intersects the stress axis, where $\dot{\varepsilon} = 0$, at a magnitude of about twice the true static yield stress, and the value of M_0 employed was based on this stress.

The permanent central deflection is overpredicted by the viscoplastic theory in the case of the aluminum targets. However, the predictions of the viscoplastic theory are in reasonable agreement with the permanent deflections of the steel plates, particularly in the lower velocity regime where strain rates are small, but their effects significant. Any discrepancies between analysis and experimental data is mainly attributable to the neglect of membrane action, particularly at the higher strain rates, as was also concluded in Ref. [8]. However, the method developed here represents a very useful tool for the analysis of large plates of a highly strain-rate dependent material such as mild steel at low and intermediate impact velocities.

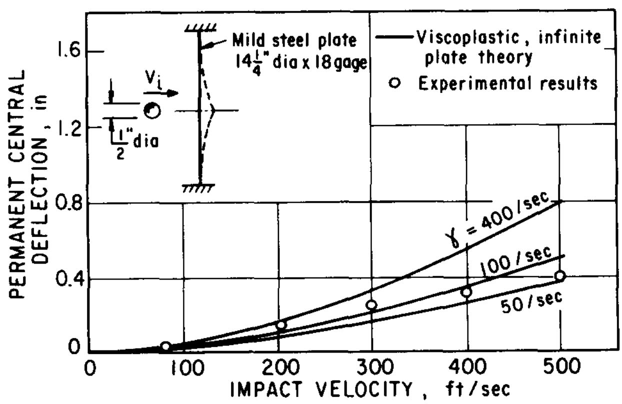

FIG. 3. Comparison of viscoplastic infinite plate theory with experimental results for projectile impact on a mild steel plate (Ref. [17]).

REFERENCES

[1] H. G. HOPKINS, On the Impact Loading of Circular Plates Made of Ductile Metal, Div. Appl. Mech., Brown Univ. Report No. DA-2598/7, AD 22938 (1953).
[2] S. J. CHULAY, Dynamic Behavior of Plastic Plates, Ph.D. Dissertation, University of California (1966).
[3] A. J. WANG, The permanent deformation of a plate under blast loading. *J. appl. Mech.* **22**, 375 (1955).
[4] H. G. HOPKINS and W. PRAGER, On the dynamics of plastic circular plates, *Z. angew. Math. Phys.* **5**, 317 (1954).
[5] A. L. FLORENCE, Clamped circular rigid–plastic plates under blast loading. *J. appl. Mech.* **33**, 256 (1966).
[6] A. L. FLORENCE, Clamped circular rigid–plastic plates under central blast loading. *Int. J. Solids Struct.* **2**, 319 (1967).
[7] A. L. FLORENCE, Circular plate under uniformly distributed impulse. *Int. J. Solids Struct.* **2**, 37 (1966).
[8] T. WIERZBICKI and A. L. FLORENCE, A theoretical and experimental investigation of impulsively loaded clamped circular viscoplastic plates. *Int. J. Solids Struct.* **6**, 553 (1970).
[9] J. M. KELLY and T. WIERZBICKI, Motion of a circular viscoplastic plate subject to projectile impact. *Z. angew. Math. Phys.* **18**, 236 (1967).
[10] J. M. KELLY and T. R. WILSHAW, A theoretical and experimental study of projectile impact on clamped circular plates. *Proc. R. Soc.* **A306**, 435 (1968).
[11] P. PERZYNA, The constitutive equations for rate sensitive plastic materials. *Q. appl. Math.* **20**, 341 (1963).
[12] J. W. CRAGGS, A rate dependent theory of plasticity. *Int. J. Engng. Sci.* **3**, 31 (1965).
[13] T. WIERZBICKI, Dynamics of rigid viscoplastic circular plates. *Arch. Mech. Stosow.* **6**, 17 (1965).
[14] M. ABRAMOWITZ and I. A. STEGUN, editors, *Handbook of Mathematical Functions*. Nat. Bur. of Stand., App. Math. Series (1964).
[15] H. DUBNER and J. ABATE, Numerical inversion of the Laplace transform and the finite Fourier cosine transform. *J. Ass. Comp. Mech.* **15**, 115 (1968).
[16] P. S. SYMONDS, Survey of methods of analysis for plastic deformation of structures under dynamic loading, Division of Engineering, Brown Univ. Report BU/NSRDC, pp. 1–67 (1967).
[17] C. A. CALDER and W. GOLDSMITH, Plastic deformation and perforation of thin plates resulting from projectile impact. *Int. J. Solids Struct.* **7**, 863 (1971).

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Абстракт—Приводится анализ для задачи бесконечной жестко вязкоупругой пластинки, подверженной действию перпендикулярного удара снарядом. Во первых получается решение в пространстве преобразования Лапласа, для конечной защемленной пластинки. Затем, путем ограничения условий и используя численные методы инверсии, получается решение для бесконечной пластинки в области времени. Результаты сравниваются с экспериментальными данными, для удара снаряда в пластинку из мягкой стали и аллюминиевого сплава. Теория представляет собой полезный аналитический метод для определения поведения боивмцх пластинок, изготовленных из материала, зависимого от скорости деформации, подверженных удару снарядом.